

Global Fit to Modified Neutrino Couplings and the Cabibbo-Angle Anomaly

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Recently, discrepancies of up to 4σ between the different determinations of the Cabibbo angle were observed. This “Cabibbo-angle anomaly” could be explained by lepton flavour universality (LFU) violating New Physics (NP) in the neutrino sector. However, modified neutrino couplings to Standard Model gauge bosons also affect many other observables sensitive to LFU violation which have to be taken into account in order to assess the viability of this explanation. Therefore, we perform a model-independent global analysis in a Bayesian approach and find that the tension in the Cabibbo angle is significantly reduced, while the agreement with other data is also mostly improved. In fact, non-zero modifications of electron and muon neutrino couplings are preferred at more than 99% CL. Still, since constructive effects in the muon sector are necessary, simple models with right-handed neutrinos (whose global fit we update as a by-product) cannot fully explain data, pointing towards more sophisticated NP models.

I. INTRODUCTION

The Standard Model (SM) of particle physics has been established with increasing precision within the last decades. In particular, both the electroweak (EW) fit [1–3] and the global CKM fit [4, 5] are in general in good agreement with the SM hypothesis and no new particles were directly observed at the LHC [6, 7]. However, there are tensions between the different determinations of the Cabibbo angle from the CKM elements V_{us} and V_{ud} which became more pronounced recently¹. Here, V_{us} from tau decays [8, 20] and V_{us} from Kaon decays [21] do not perfectly agree. Furthermore, there is a $\sim 3-4\sigma$ tension of these determinations with the one from V_{ud} entering super-allowed β decay (using CKM unitarity) with non-negligible dependence on the theory predictions [22–25]. In more detail, the different determinations of V_{us} are:

- Measurements of $K \rightarrow \pi \ell \nu$ together with the form factor $f_+(0)$ evaluated at zero momentum transfer result in $V_{us} = 0.2232(11)$ [21].
- $K \rightarrow \ell \nu / \pi \rightarrow \ell \nu$ determines V_{us}/V_{ud} once the ratio of decay constants f_{K^\pm}/f_{π^\pm} is known. Using CKM unitarity this results in $V_{us} = 0.22534(44)$ [21].
- V_{ud} is measured via super-allowed nuclear β decay. Here V_{us} is again determined via CKM unitarity and using the theory input of Marciano et al. [25]

one finds $V_{us} = 0.22699(77)$, while the evaluation of Seng et al. gives $V_{us} = 0.22780(59)$ [23].

- V_{us}/V_{ud} is also measured in $\tau \rightarrow K \nu$, $\tau \rightarrow K \nu / \tau \rightarrow \pi \nu$ and via inclusive tau decays. Here the HFLAV average is $V_{us} = 0.2221(13)$ [8].

This situation is graphically depicted in Fig. 1. One can clearly see that these measurements are not consistent with each other, and Ref. [26] quantifies this inconsistency to be at the level of 3.6σ (5.1σ) if the theory input of Ref. [25] (Ref. [23]) for super-allowed beta decay is used.

Therefore, it is very interesting to explore if NP can explain this “Cabibbo-angle anomaly”. First of all, note that the absolute size of a NP effect potentially capable of explaining this anomaly is quite large since the corresponding SM contribution is generated at tree-level and is at most suppressed by one power of the Wolfenstein parameter. Because of this, at the level of effective operators, and given the strong LHC bounds on NP generating 2-quark-2-lepton operators [27], NP entering via 4-fermion operators seems to be a disfavoured option. Another possibility is a modification of W -fermion couplings, where a right-handed W -coupling to quarks only improves the fit mildly [26]. Furthermore, a modification of left-handed W -couplings to quarks (which is equivalent to an apparent violation of CKM unitarity) can improve the agreement between super-allowed beta decay and V_{us} from Kaon decays [28], but generates potentially dangerous effects in other flavour observables (like Kaon mixing). Therefore, we will follow a different avenue in this letter and study the impact of modified W -boson couplings to neutrinos.

Modified couplings of neutrinos to the SM W are generated via higher dimensional operators in an EFT approach. Here, due to $SU(2)_L$ gauge invariance, in general not only W -neutrino couplings but also Z -neutrino couplings are modified. Moreover, these modified cou-

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¹ There are also intriguing hints for the violation of lepton flavour universality (LFU) in semi-leptonic B decays, both in the $b \rightarrow c \tau \nu$ [8] and $b \rightarrow s \ell^+ \ell^-$ [9–19] transitions (at the $3-5\sigma$ level) which could be related within a UV complete model to the observables considered in this article.

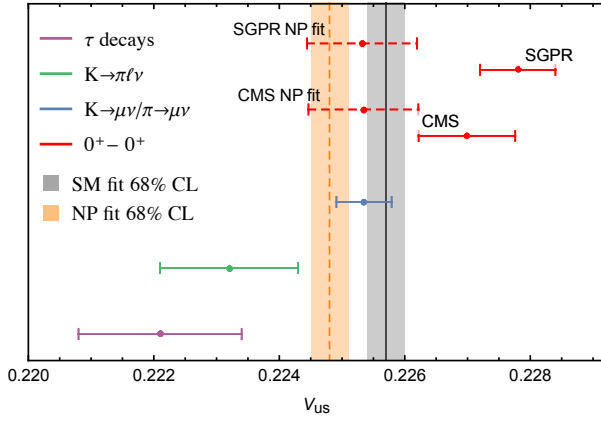


FIG. 1. Measurements of V_{us} from τ decays, $K \rightarrow \pi l \nu$, $K \rightarrow \mu \nu / \pi \rightarrow \mu \nu$, and $0^+ - 0^+$ transition using CKM unitarity to convert V_{ud} to V_{us} . The grey band shows the 68% CL posterior within the SM while the orange band corresponds to the NP fit with non-zero values of ε_{ii} . Here SGPR (CMS) stands for the V_{us} values extracted from super-allowed beta decays using the theory input of Ref. [22] (Ref. [25]). Accidentally, the posterior of V_{us} is the same, independently of the theory input used for beta decays (to the numerical accuracy at which we are working). The effect of ε_{ii} on V_{us} within the NP fit is quite small except for the determination from super-allowed beta decay. Note that V_{us} of the NP fit corresponds to the one of the Lagrangian, defined within a unitary CKM matrix.

plings not only enter Z and W decays, but also all low energy observables involving neutrinos. In particular, ratios testing lepton flavour universality (LFU) in K , π and W decays are most relevant due to their exquisite experimental and theoretical precision. There are stringent bounds from $K \rightarrow \mu \nu / K \rightarrow e \nu$ [29, 30], $\pi \rightarrow \mu \nu / \pi \rightarrow e \nu$ [31, 32] as well as from $\tau \rightarrow \mu \nu \nu / \tau \rightarrow e \nu \nu$ or $W \rightarrow \mu \nu / W \rightarrow e \nu$ [33]. Correlated effects arise, and it is clear that a global fit to all data is necessary in order to assess consistently the impact of modified neutrino couplings.

Modified neutrino couplings to SM gauge bosons have already been considered in the literature in the context of right-handed neutrinos [34–59] and global fits have also been performed [60, 61]. However, extensions with right-handed neutrinos lead necessarily to destructive interference with the SM, whereas here we will also be interested in the most general case allowing for an arbitrary phase of the NP contribution. The connection to, and correlations with, the Cabibbo-angle anomaly were not considered before and, in addition, we will use the publicly available `HEPfit` software [62] to perform a Bayesian analysis, while previous analyses were based in frequentist inference.

After defining our setup and reviewing the relevant observables together with the corresponding NP modification in the following section II, we will present the results of our fit in Sec. III before we conclude in Sec. IV.

II. SETUP AND OBSERVABLES

As outlined in the introduction, we want to assess the impact of modified neutrino couplings to gauge bosons within an EFT approach. For this purpose, we assume that the NP scale is above the EW scale, as suggested by LHC [6, 7] and LEP [63] searches. Therefore, NP interactions must be $SU(2)_L$ gauge invariant and the number of operators is significantly reduced [64]. In fact, at the dimension 6 level, there is just one operator which modifies only the couplings of gauge bosons to neutrinos [64, 65]:

$$\bar{L}_i \gamma^\mu \tau^I L_j H^\dagger i \overleftrightarrow{D}_\mu^I H \quad \text{with } \tau^I = (1, -\sigma_1, -\sigma_2, -\sigma_3), \quad (1)$$

where σ_i are the Pauli matrices². Note that this operator is Hermitian, meaning the diagonal elements are real. In what follows, we conveniently parametrize the effect of a non-zero Wilson coefficient of this operator in such a way that a neutrino entering a gauge coupling carries a (small) modification of $\frac{1}{2}\varepsilon_{ij}$, resulting in shifts in the W and Z Feynman rules,

$$\begin{aligned} \frac{-ig_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L \nu_j W_\mu &\Rightarrow \frac{-ig_2}{\sqrt{2}} \bar{\ell}_i \gamma^\mu P_L \nu_j W_\mu \left(\delta_{ij} + \frac{1}{2}\varepsilon_{ij} \right), \\ \frac{-ig_2}{2c_W} \bar{\nu}_i \gamma^\mu P_L \nu_j Z_\mu &\Rightarrow \frac{-ig_2}{2c_W} \bar{\nu}_i \gamma^\mu P_L \nu_j Z_\mu (\delta_{ij} + \varepsilon_{ij}). \end{aligned} \quad (2)$$

Here we assumed massless neutrinos and thus suppressed the PMNS matrix in the W vertex. In the subsections below we look at observables that are changed by the modifications of these couplings, and which will consequently be included in the global fit. We will review the experimental situation and quantify how the observables are affected by the modified neutrino couplings.

A. Lepton Flavour Violating Decays

Non-diagonal elements of ε_{ij} lead to charged lepton flavour violation. Here the bounds from radiative lepton decays $\ell_i \rightarrow \ell_f \gamma$ are most stringent. Using the results of Ref. [66] we obtain

$$\text{Br}[\ell_i \rightarrow \ell_f \gamma] = \frac{m_{\ell_i}^3}{4\pi\Gamma_{\ell_i}} |c_R^{\ell_f \ell_i}|^2, \quad (3)$$

with

$$c_R^{\ell_f \ell_i} = -\frac{10e}{384\pi^2 M_W^2} \frac{g_2^2}{2} m_{\ell_i} \varepsilon_{fi}, \quad (4)$$

² I.e. it is the difference of the two operators $Q_{\varphi\ell}^{(1)}$ and $Q_{\varphi\ell}^{(3)}$ in the basis of Ref. [65], to which we refer the interested reader for details on the conventions.

Observable	Ref.	Measurement
$\frac{K \rightarrow \mu\nu}{K \rightarrow e\nu} \simeq 1 + \frac{1}{2}\varepsilon_{\mu\mu} - \frac{1}{2}\varepsilon_{ee} $	[69]	0.9978 ± 0.0020
$\frac{\pi \rightarrow \mu\nu}{\pi \rightarrow e\nu} \simeq 1 + \frac{1}{2}\varepsilon_{\mu\mu} - \frac{1}{2}\varepsilon_{ee} $	[31, 32]	1.0010 ± 0.0009
$\frac{\tau \rightarrow \mu\nu\bar{\nu}}{\tau \rightarrow e\nu\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\mu\mu} - \frac{1}{2}\varepsilon_{ee} $	[8, 32]	1.0018 ± 0.0014
$\frac{K \rightarrow \pi\mu\bar{\nu}}{K \rightarrow \pi e\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\mu\mu} - \frac{1}{2}\varepsilon_{ee} $	[69]	1.0010 ± 0.0025
$\frac{W \rightarrow \mu\bar{\nu}}{W \rightarrow e\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\mu\mu} - \frac{1}{2}\varepsilon_{ee} $	[69, 70]	0.996 ± 0.010
$\frac{\tau \rightarrow e\nu\bar{\nu}}{\mu \rightarrow e\nu\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\tau\tau} - \frac{1}{2}\varepsilon_{\mu\mu} $	[8, 32]	1.0010 ± 0.0014
$\frac{\tau \rightarrow \pi\nu}{\pi \rightarrow \mu\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\tau\tau} - \frac{1}{2}\varepsilon_{\mu\mu} $	[8]	0.9961 ± 0.0027
$\frac{\tau \rightarrow K\nu}{K \rightarrow \mu\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\tau\tau} - \frac{1}{2}\varepsilon_{\mu\mu} $	[8]	0.9860 ± 0.0070
$\frac{W \rightarrow \tau\bar{\nu}}{W \rightarrow \mu\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\tau\tau} - \frac{1}{2}\varepsilon_{\mu\mu} $	[69, 70]	1.034 ± 0.013
$\frac{\tau \rightarrow \mu\nu\bar{\nu}}{\mu \rightarrow e\nu\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\tau\tau} - \frac{1}{2}\varepsilon_{ee} $	[8, 32]	1.0029 ± 0.0014
$\frac{W \rightarrow \tau\bar{\nu}}{W \rightarrow e\bar{\nu}} \simeq 1 + \frac{1}{2}\varepsilon_{\tau\tau} - \frac{1}{2}\varepsilon_{ee} $	[69, 70]	1.031 ± 0.013
$\frac{B \rightarrow D^{(*)}\mu\nu}{B \rightarrow D^{(*)}e\nu} \simeq 1 + \frac{1}{2}\varepsilon_{\mu\mu} - \frac{1}{2}\varepsilon_{ee} $	[71]	0.989 ± 0.012

TABLE I. Ratios testing LFU together with their dependence on the modifications of the neutrino couplings ε_{ij} . Here it is implicitly understood that these ratios are normalized to their values within the SM, such that any deviation from unity measures LFU violation.

where we keep only linear terms in ε_{if} and neglect the small mass of the outgoing lepton. The current experimental limits for lepton flavour violation processes are [67, 68]:

$$\begin{aligned} \text{Br}[\mu \rightarrow e\gamma] &\leq 4.2 \times 10^{-13}, \\ \text{Br}[\tau \rightarrow \mu\gamma] &\leq 4.4 \times 10^{-8}, \\ \text{Br}[\tau \rightarrow e\gamma] &\leq 3.3 \times 10^{-8}, \end{aligned} \quad (5)$$

at 90% CL and result in to the following limits:

$$\begin{aligned} |\varepsilon_{e\mu}| &\leq 1.4 \times 10^{-5}, \\ |\varepsilon_{\mu\tau}| &\leq 9.4 \times 10^{-3}, \\ |\varepsilon_{e\tau}| &\leq 1.1 \times 10^{-2}. \end{aligned} \quad (6)$$

These limits on the flavour off-diagonal elements can be used directly, as they are unaffected (at leading order in ε_{ij}) by other entries ε_{ij} . Furthermore, since flavour off-diagonal elements of ε_{ij} in flavour conserving processes do not interfere with the SM contributions, they enter only quadratically. Given this, in conjunction with the strict bounds derived above, ε_{ij} with $i \neq j$ can be safely neglected in these processes, which we consider henceforth.

B. EW Observables

Measurements of the EW observables, as performed at LEP [70, 72], are high precision tests of the SM. The EW sector of the SM can be completely parametrized by the three Lagrangian parameters v , g_1 and g_2 ; then, other quantities like G_F , m_W or m_Z can be expressed in

terms of these parameters and their measurements allow for consistency tests. However, for practical purposes it is better to choose another set of three parameters parametrizing the EW sector of the SM: a convenient choice is to use the quantities with the smallest experimental error of their direct measurements, i.e. the mass of the Z boson (m_Z), the Fermi constant (G_F) and the fine structure constant (α).

While the measurements of m_Z and α are not affected by the modification of the neutrino couplings in Eq. (2), G_F (which is determined with a very high precision from the muon lifetime) is. As such, its value, extracted from $\mu \rightarrow e\nu\nu$, depends on the modification of the W - ℓ - ν coupling. Taking into account that $\text{Br}(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu) \sim 1$ we have

$$\frac{1}{\tau_\mu} = \frac{(G_F^{\mathcal{L}})^2 m_\mu^5}{192\pi^3} (1 + \Delta q) \left(1 + \frac{1}{2}\varepsilon_{ee} + \frac{1}{2}\varepsilon_{\mu\mu}\right)^2 \quad (7)$$

where $G_F^{\mathcal{L}}$ is the Fermi constant appearing in the Lagrangian and Δq includes phase space, QED and hadronic radiative corrections. Thus we find

$$G_F = G_F^{\mathcal{L}} \left(1 + \frac{1}{2}\varepsilon_{ee} + \frac{1}{2}\varepsilon_{\mu\mu}\right). \quad (8)$$

In our analysis, it is convenient that we use G_F , not $G_F^{\mathcal{L}}$, as a fit parameter whose prior is determined by its direct measurement $1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ [73]. Note that $G_F^{\mathcal{L}}$ enters all other EW observables which are thus indirectly modified by ε_{ii} entering in Eq. (8). These observables, computed from G_F , m_Z and α , include m_W , the hadronic Z -pole cross-section (σ_h^0), and the leptonic vector and axial-vector couplings, g_V^ℓ and g_A^ℓ . Since the Z sector remains lepton flavour universal (for charged leptons) we can thus use the five standard Z observables (assuming LFU) [72]: m_Z , Γ_Z , σ_{had}^0 , R_ℓ^0 , and $A_{\text{FB}}^{0,\ell}$. In the fits we make, we trade R_ℓ^0 and $A_{\text{FB}}^{0,\ell}$ for g_V^ℓ and g_A^ℓ . The Higgs mass (m_h), the top mass (m_t) and the strong coupling constant (α_s) have to be included as fit parameters as well, since they enter indirectly EW observables via loop effects.

In addition to G_F , only the total width of the Z (Γ_Z) and the number of light neutrino extracted from invisible Z decays (N_ν) receive direct modifications in the presence of anomalous neutrino couplings. Here the number of active neutrinos, as extracted from data [72], is given by

$$\begin{aligned} N_\nu^{\text{exp}} &= (1 + \varepsilon_{ee})^2 + (1 + \varepsilon_{\mu\mu})^2 + (1 + \varepsilon_{\tau\tau})^2 \\ &= 2.9840 \pm 0.0082, \end{aligned} \quad (9)$$

which in turn also changes Γ_Z , to which it contributes.

C. Test of LFU

In case the diagonal elements of ε_{ii} differ from each other, observables testing LFU provide stringent constraints. Here, we have ratios of W decays ($W \rightarrow$

Parameter	Prior	SM posterior
G_F [GeV ⁻²] [32]	$1.1663787(6) \times 10^{-5}$	★
α [32]	$7.2973525664(17) \times 10^{-3}$	★
$\alpha_s(M_Z)$ [32]	0.1181(11)	0.1179(11)
m_Z [GeV] [72]	91.1875 ± 0.0021	91.1885 ± 0.0020
m_H [GeV] [74, 75]	125.16 ± 0.13	★
m_t [GeV] [76–78]	172.80 ± 0.40	173.00 ± 0.39

TABLE II. Parameters of the EW fit together with their (Gaussian) priors and posteriors. Here, the posteriors that remain equal to the prior (up to the numerical accuracy considered) are abbreviated by a star. Furthermore, once the effect of ε_{ij} is included, only the posterior of the top (pole) mass changes slightly to $m_t \sim 172.8$ while all other parameters remain the same, showing the EW fit is, to a very good approximation, independent of the flavour parameters shown in Fig. III.

$\ell_i \nu / W \rightarrow \ell_j \nu$) as well as of Kaon, pion and tau decays (see Ref. [69] for an overview). Concerning B decays, only $B \rightarrow D^{(*)} e \nu / B \rightarrow D^{(*)} \mu \nu$ provides a relevant constraint [71]. The corresponding observables, including their dependence on ε_{ii} are shown in Table I. For tau decays, we include their correlations as given in Ref. [8].

D. Determination of $|V_{us}|$

We can now turn to the determination of V_{us} as already briefly depicted in the introduction (see Fig. 1).

$K_{\ell 3}$: V_{us} can be determined from the semi-leptonic Kaon decays. In order to allow for LFU violation, one has to separate muon from electron modes. Averaging K_L , K^\pm and K_S modes [32], one finds

$$\begin{aligned} |V_{us}^{K\mu 3}| &\simeq 0.2234(8), \\ |V_{us}^{Ke 3}| &\simeq 0.2230(21), \end{aligned} \quad (10)$$

by using the lattice average [21] of the form factor at zero momentum transfer

$$f_+(0) = 0.9698(17) \quad (N_f = 2 + 1 + 1). \quad (11)$$

We choose to include the muon mode in the global fit, while the electron mode is already taken into account via the LFU ratios in Table. I. The NP modification, including the indirect effect of G_F , is:

$$|V_{us}^{K\mu 3}| \simeq |V_{us}^{\mathcal{L}}| \left(1 - \frac{1}{2} \varepsilon_{ee} \right) \quad (12)$$

$K_{\ell 2}$: $\text{Br}(K^\pm \rightarrow \mu^\pm \nu) / \text{Br}(\pi^\pm \rightarrow \mu^\pm \nu)$ determines V_{us}/V_{ud} . Including long-distance electromagnetic and strong isospin breaking corrections [79] and using the average of the lattice determinations for the ratio of form factors [21],

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.1967(18) \quad (N_f = 2 + 1 + 1), \quad (13)$$

	Prior	NP-I posterior	NP-II posterior
$V_{us}^{\mathcal{L}}$	0.225 ± 0.010	0.2248 ± 0.0004	0.2248 ± 0.0004
ε_{ee}	0.00 ± 0.05	-0.0027 ± 0.0007	-0.0030 ± 0.0007
$\varepsilon_{\mu\mu}$	0.00 ± 0.05	0.0008 ± 0.0004	0.0012 ± 0.0003
$\varepsilon_{\tau\tau}$	0.00 ± 0.05	-0.0011 ± 0.0018	-0.0010 ± 0.0018

TABLE III. Fit parameters of the flavour sector together with their priors and posteriors for the two NP scenarios. One can see that the choice between the two V_{us} determinations has a small effect on ε_{ee} and $\varepsilon_{\tau\tau}$, while in scenario NP-I a vanishing value of $\varepsilon_{\mu\mu}$ is more compatible with data (within $\approx 2 \sigma$). Note that the value for $V_{us}^{\mathcal{L}}$ is accidentally the same (to the numerical accuracy given): the effect of the different V_{us} determinations is compensated by the difference in the preferred regions for ε_{ii} . However, the deviation from the SM posterior, 0.2257(3), is indeed sizable (see Fig.1).

we find

$$|V_{us}^{K/\pi}| \simeq 0.22535(44). \quad (14)$$

where we assumed CKM unitarity and took $|V_{ub}| \simeq 0.004$. Note that the value of V_{us} is very insensitive to V_{ub} , whose uncertainty can therefore be neglected, and that this V_{ub} determination is not affected by ε_{ij} .

$0^+ - 0^+$ transitions: $|V_{ud}|$ can be extracted from super-allowed nuclear β transitions [80]. The result relies heavily on the evaluation of radiative corrections. We consider the two different results (as suggested in Ref. [26]) of Marciano et al. [25] (CMS) and Seng et al. [23] (SGPR), which give

$$\begin{aligned} |V_{us}|_{\text{CMS}} &= 0.22699(77), \\ |V_{us}|_{\text{SGPR}} &= 0.22780(60), \end{aligned} \quad (15)$$

where we used again CKM unitarity. Turning on the NP couplings, we find the following modification

$$|V_{us}^{\beta}| \simeq \sqrt{1 - |V_{ud}^{\mathcal{L}}|^2 \left(1 - \frac{1}{2} \varepsilon_{\mu\mu} \right)^2}. \quad (16)$$

τ decays: $|V_{us}|$ can be also determined from hadronic τ decays [8]. Here the average is [8]

$$|V_{us}^{\tau}| = 0.2221 \pm 0.0013. \quad (17)$$

Both $\tau \rightarrow K \nu / \tau \rightarrow \pi \nu$ and the inclusive mode measure V_{us}/V_{ud} , which means there is, at leading order, no dependence on ε_{ij} , and the determination is then unaffected by our NP contributions. This is different for the determination from $\tau \rightarrow K \nu$, whose dependence on ε is given by

$$|V_{us}^{\tau \rightarrow K \nu}| \simeq |V_{us}^{\mathcal{L}}| \left(1 - \frac{1}{2} \varepsilon_{ee} - \frac{1}{2} \varepsilon_{\mu\mu} + \frac{1}{2} \varepsilon_{\tau\tau} \right). \quad (18)$$

Since this mode as well as other hadronic tau decays are already included in the LFU ratios, we do not include the V_{us} from tau decays in our global fit. Nevertheless, we can still predict the change in $V_{us}^{\tau \rightarrow K \nu}$.

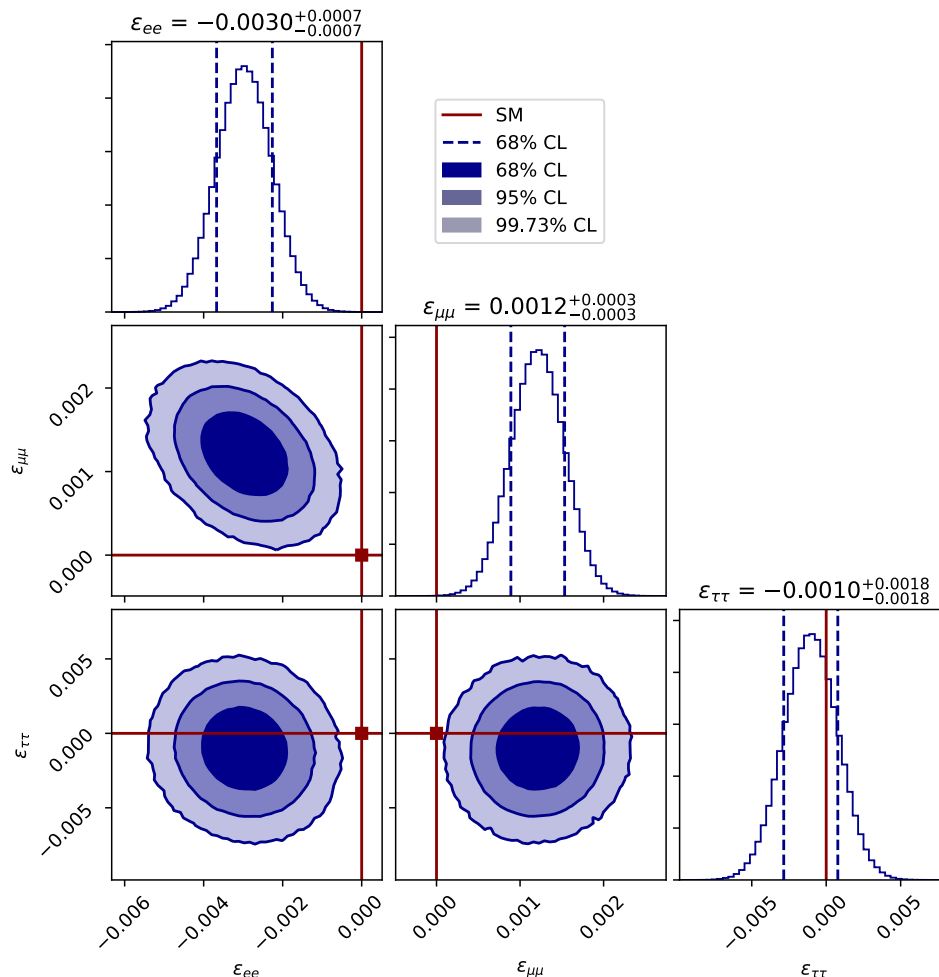


FIG. 2. Global fit for scenario NP-II. The 2-D fit (68%, 95% and 99% CL) for $\varepsilon_{ii}-\varepsilon_{jj}$ as well as the 1-D fit for each ε_{ii} (68% CL indication) are shown. Note that from Table IV one can see that the fit for scenario NP-I will look very similar, even though the regions will be a bit closer ($\approx 1\sigma$) to the SM point.

III. ANALYSIS

In this section we perform the global fit to the modified neutrino couplings (see Eq. (2)), taking into account the observables discussed in the previous section. Before presenting the results, let us briefly discuss the statistical inference procedure we adopted. Our analysis is performed in a Bayesian framework using the publicly available `HEPfit` package [62], whose Markov Chain Monte Carlo (MCMC) determination of posteriors is powered by the Bayesian Analysis Toolkit (BAT) [81].

The fit parameters of the EW sector are reported in Table II while the parameters of the flavour sector are given in Table III. For G_F , α , α_s , M_Z , m_t and m_H we assume Gaussian priors, which correspond to the current direct measurements or evaluations of these parameters, whereas $V_{us}^{\mathcal{L}}$ and ε_{ii} have flat priors. We verified that the chosen ranges of flat priors yield well-determined probability density functions (*p.d.f.*), i.e. they are chosen in such a way that larger ranges would not significantly alter

our results ³.

In order to not overweight the V_{ud} measurements from $0^+ - 0^+$ transitions, we do not include both theory determinations at the same time, but rather define two scenarios:

NP-I : $|V_{ud}|_{\text{CMS}}$ from Ref. [25],

NP-II : $|V_{ud}|_{\text{SGPR}}$ from Ref. [22].

Bayesian model comparison between different scenarios can be accomplished by evaluating an *Information Criterion* (IC) [82, 83]. This quantity is characterized by the mean and the variance of the posterior of the log-likelihood, $\log \mathcal{L}$, which yield an estimate of the predictive accuracy of the model [84], and a penalty factor for the

³ Employing the Metropolis-Hastings algorithm implemented in BAT to sample from the posterior distribution, our MCMC runs involved 6 chains with a total of 2 million events per chain, collected after an equivalent number of pre-run iterations.

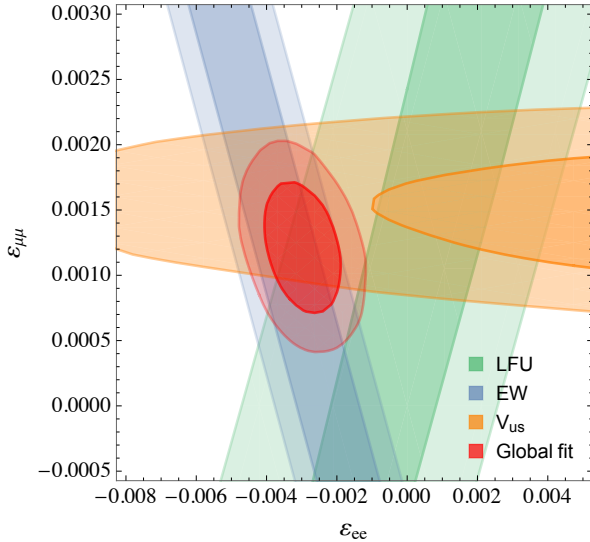


FIG. 3. Contributions to the global fit from different classes of observables: LFU (green), EW (blue) and V_{us} (orange). The global fit (red) is superimposed. Dark and light colors correspond to the 68% CL and 95% CL, respectively.

number of free parameters fitted. Preference for a model is given according to the smallest IC value, following the scale of evidence suggested in Refs. [85, 86].

First, we redo the global EW fit within the SM whose results for the fit parameters are given in Table IV⁴. Note that their posteriors remain practically unchanged once NP effects via ε_{ij} are included⁵. This is, however, not the case for V_{us} where the SM value of 0.2257(3) significantly changes once NP is included. Note that the main tensions within the SM originate from $Z \rightarrow \nu\nu$, g_V^ℓ , and the conflicting determinations of V_{us} from $K_L \rightarrow \pi\mu\nu$ and super-allowed beta decays. These are also the observables where most significant improvement is achieved in the NP case while even though there is a tension in $W \rightarrow \tau\nu/W \rightarrow e\nu$, this cannot be resolved in our NP scenario.

Therefore, let us now probe the impact of non-zero values of ε_{ij} . As noted in the last section, one can neglect the flavour off-diagonal elements whose values are directly bounded by radiative lepton decays. As such, in the global fit we only have to consider ε_{ee} , $\varepsilon_{\mu\mu}$ and $\varepsilon_{\tau\tau}$. The 68% CL intervals for fit parameters of the flavour sec-

tor ($V_{us}^\mathcal{L}$, ε_{ii}) within the two NP scenarios can be found in Table III. One can see that there is only a mild difference between both scenarios. In particular, the posterior of $V_{us}^\mathcal{L}$ is accidentally even the same and only the preferred region for $\varepsilon_{\mu\mu}$ in scenarios NP-II is more positive than in scenario NP-I. Therefore, we only present the results for the two dimensional ε_{ii} - ε_{jj} planes in Fig. 2 within scenario NP-II (scenario NP-I is approximately 1σ more compatible with the SM hypothesis). There, the 68%, 95% and 99% CL contours are shown, and it is clear from the ε_{ee} - $\varepsilon_{\mu\mu}$ plane, where the largest deviation from SM can be found, that these regions do not overlap with the SM point $\varepsilon_{ii} = 0$, and that ε_{ee} and $\varepsilon_{\mu\mu}$ possess an anti-correlation. This interesting fit in the ε_{ee} - $\varepsilon_{\mu\mu}$ plane is depicted in more detail in Fig. 3 where, in addition, the preferred regions from the individual classes of processes are shown. The fact that all regions overlap at the 68% CL reflects the goodness of the fit.

In order to better judge the agreement of the NP hypotheses with data and how this compares to the SM, we define the pull (with respect to the SM) for an observable O_i as

$$P(O_i) = \left| \frac{O_i^{\text{exp}} - O_i^{\text{SM}}}{\sqrt{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{SM}})^2}} \right| - \left| \frac{O_i^{\text{exp}} - O_i^{\text{NP}}}{\sqrt{(\sigma_i^{\text{exp}})^2 + (\sigma_i^{\text{NP}})^2}} \right|. \quad (19)$$

These pulls are reported in Table IV for the two NP scenarios, where also the SM and NP posteriors for all observables (except the EW fit parameters shown in Fig. II which are unaffected by the NP parameters) are shown. Pulls signalling a better agreement between NP and data (with respect to the SM) are highlighted in gray. Concerning the V_{us} determination, we also depicted the posterior of scenario NP-II and the value extracted from super-allowed beta decay in Fig. 1. From this, one can see that the main drivers leading to a better fit of the NP scenarios are the V_{us} determination from super-allowed beta decays, g_V^ℓ and $\tau \rightarrow \mu\nu\nu/\tau \rightarrow e\nu\nu$.

For a more direct model comparison between the NP hypothesis and the SM we look at the IC values. Here we obtain for the SM $\text{IC}_{\text{SM}} = 70$, compared to $\text{IC}_{\text{NP-II}} = 56$ and $\text{IC}_{\text{NP-I}} = 55$ for the two NP scenarios. In the vein of Ref. [86], this constitutes “very strong” evidence against the SM, further evidencing that current data clearly favours the NP hypothesis, and promoting the search for a NP model.

For a UV complete NP explanation obviously the possibility of right-handed neutrinos comes to mind, as these models give tree-level effects in $Z\nu\nu$ and $W \rightarrow \ell\nu$ couplings. However, here the effect is necessarily destructive (i.e. $\varepsilon_{ii} < 0$), which is not compatible with the preferred regions found in our fit. Nevertheless, since these models are well motivated by the observed non-zero neutrino masses, we present the constrained fit with negative values of ε_{ii} in Fig. 4. Even though the MCMC is able to converge and provide regions compatible with data, the shift towards values compatible with zero (within $\sim 2.8\sigma$

⁴ Concerning the W mass computation, `HEPfit` provides both the option of using the recent precise numerical formula from Ref. [87], and the usual determination of M_W from the Z -boson mass, α , and G_F [88], with radiative corrections encoded in Δr (which has been known up to 3 loop $\mathcal{O}(\alpha^3)$ EW [89] and $\mathcal{O}(\alpha\alpha_s^2, \alpha^2\alpha_s)$ EW-QCD contributions [89–92]). Due to the direct modifications of G_F under analysis, we opted for the latter.

⁵ This is the case for G_F (due to its precise measurement) which is used as a fit parameter, yet not for $G_F^\mathcal{L}$ whose posterior changes to $(1.16743 \pm 0.00044) \times 10^{-5}$ and $(1.16749 \pm 0.00047) \times 10^{-5}$ within scenario NP-I and NP-II, respectively.

Observable	Ref.	Measurement	SM Posterior	NP-I posterior	NP-II posterior	Pull I	Pull II
m_W [GeV]	[3, 32]	80.373 ± 0.020	80.363 ± 0.0036	80.3766 ± 0.0060	80.3756 ± 0.0060	0.32	0.36
Γ_Z [GeV]	[72]	2.4952 ± 0.0023	2.49457 ± 0.00059	2.4966 ± 0.0012	2.4964 ± 0.0012	-0.27	-0.20
σ_h^0 [nb]	[72]	41.541 ± 0.037	41.4856 ± 0.0058	41.518 ± 0.021	41.516 ± 0.021	0.94	0.80
g_V^ℓ	[72]	-0.03783 ± 0.00041	-0.036763 ± 0.000060	-0.03734 ± 0.00021	-0.03730 ± 0.00021	1.54	1.45
N_ν^{exp}	[72]	2.9840 ± 0.0082	3	2.9940 ± 0.0038	2.9944 ± 0.0038	0.84	0.80
$\frac{K \rightarrow \mu \nu}{K \rightarrow e \nu}$	[29, 30, 69, 103]	0.9978 ± 0.0020	1	1.00171 ± 0.00047	1.00209 ± 0.00044	-0.80	-0.99
$\frac{\pi \rightarrow \mu \nu}{\pi \rightarrow e \nu}$	[31, 32, 103–106]	1.0010 ± 0.0009	1	1.00171 ± 0.00047	1.00209 ± 0.00044	0.41	0.02
$\frac{\tau \rightarrow \mu \nu}{\tau \rightarrow e \nu}$	[8, 32]	1.0018 ± 0.0014	1	1.00171 ± 0.00047	1.00209 ± 0.00044	1.22	1.10
$\frac{K \rightarrow \pi \mu \nu}{K \rightarrow \pi e \nu}$	[69, 107, 108]	1.0010 ± 0.0025	1	1.00171 ± 0.00047	1.00209 ± 0.00044	0.12	-0.03
$\frac{W \rightarrow \mu \nu}{W \rightarrow e \nu}$	[69, 70]	0.996 ± 0.010	1	1.00171 ± 0.00047	1.00209 ± 0.00044	-0.17	-0.21
$\frac{\tau \rightarrow e \nu}{\mu \rightarrow e \nu}$	[8, 32]	1.0010 ± 0.0014	1	0.99907 ± 0.00093	0.99888 ± 0.00092	-0.43	-0.55
$\frac{\tau \rightarrow \pi \nu}{\pi \rightarrow \mu \nu}$	[8]	0.9961 ± 0.0027	1	0.99907 ± 0.00093	0.99888 ± 0.00092	0.40	0.47
$\frac{\tau \rightarrow K \nu}{K \rightarrow \mu \nu}$	[8]	0.9860 ± 0.0070	1	0.99907 ± 0.00093	0.99888 ± 0.00092	0.15	0.18
$\frac{W \rightarrow \tau \nu}{W \rightarrow \mu \nu}$	[69, 70]	1.034 ± 0.013	1	0.99907 ± 0.00093	0.99888 ± 0.00092	-0.06	-0.08
$\frac{\tau \rightarrow \mu \nu}{\mu \rightarrow e \nu}$	[8, 32]	1.0029 ± 0.0014	1	1.0008 ± 0.0010	1.0010 ± 0.0010	0.85	0.97
$\frac{W \rightarrow \tau \nu}{W \rightarrow e \nu}$	[69, 70]	1.031 ± 0.013	1	1.0008 ± 0.0010	1.0010 ± 0.0010	0.07	0.08
$\frac{B \rightarrow D^{(*)} \mu \nu}{B \rightarrow D^{(*)} e \nu}$	[71]	0.989 ± 0.012	1	1.0017 ± 0.0005	1.0021 ± 0.0004	-0.14	-0.17
$ V_{us}^{K\mu 3} $	[21, 32]	0.2234 ± 0.0008	0.2257 ± 0.0003	0.22507 ± 0.00039	0.22514 ± 0.00039	0.77	0.69
$ V_{us}/V_{ud} ^{K/\pi}$	[21, 79]	0.2313 ± 0.0005	0.23168 ± 0.00036	0.23068 ± 0.00043	0.23072 ± 0.00043	-0.32	-0.26
$ V_{ud}^\beta _{\text{CMS}}$	[25, 80]	0.97389 ± 0.00018	0.974196 ± 0.000078	0.97404 ± 0.00017	-	0.95	-
$ V_{ud}^\beta _{\text{SGPR}}$	[23, 80]	0.97370 ± 0.00014	0.974196 ± 0.000078	-	0.97381 ± 0.00013	-	2.52

TABLE IV. Observables included in and predicted by our global fit. Here the SM scenario, as well as the NP-I and NP-II hypotheses are shown. The pulls are defined with respect to the SM in such a way that positive values means better agreement of the NP hypothesis (highlighted in grey). Note that the observables testing LFU are implicitly normalized in such a way that they correspond to unity within the SM.

for ε_{ee} , and 1.5σ for $\varepsilon_{\tau\tau}$) signals a feeble improvement with respect to the SM. In fact, such a conclusion is supported by an IC value of 71, which is even slightly bigger than the one of the SM due to the penalty for extra parameters. Moreover, once the constraint from $\mu \rightarrow e\gamma$, arising in models with $\varepsilon_{e\mu} = \sqrt{\varepsilon_{ee}\varepsilon_{\mu\mu}}$ [55, 95–102] is taken into account, it is even more difficult in this scenario to explain data well.

IV. CONCLUSIONS

In this article we performed a model independent global fit to modified neutrino couplings motivated by the Cabbibo-angle anomaly (i.e. the disagreement between the different determinations of V_{us}). Taking into account all relevant observables related to the EW sector of the SM and observables testing LFU (like $\tau \rightarrow \mu\nu\nu/\tau \rightarrow e\nu\nu$, $\pi \rightarrow \mu\nu\nu/\pi \rightarrow e\nu\nu$, etc.), we found that agreement with data can be significantly improved by small modifications ε_{ii} . Our results for this NP scenario are depicted in

Fig. 2, showing the SM hypothesis lies well beyond the 99% CL region. Furthermore, the IC values of the scenarios here considered strongly prefer the NP hypothesis.

However, conventional models with right-handed neutrinos, which lead to necessary destructive interference, cannot explain data very well. Nevertheless, since these models are well motivated by the observed non-vanishing neutrino masses, we updated their global fit, taking into account the different V_{us} determinations, in Fig. 4.

Clearly, more data and further theory input is needed to clarify the situation in the future. Also, the study/construction of NP models which can give a constructive effect in Z - ν - ν and W - ℓ - ν couplings is a promising direction of research, building upon the results of this article.

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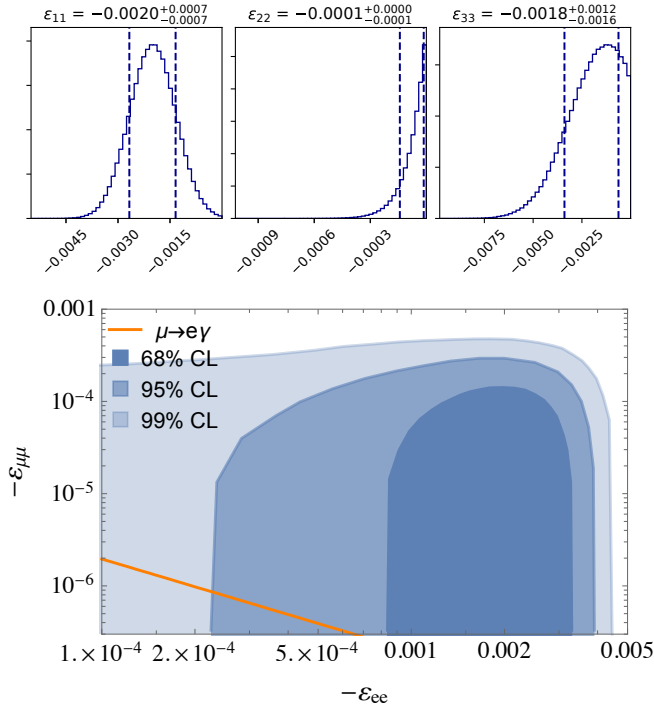


FIG. 4. Global fit for scenario NP-II confining ε_{ii} to be negative, as obtained in models with effective PMNS unitarity violation from mixing with right-handed neutrinos [47, 55, 93, 94] and considered in previous fits [60, 61]. In addition, we show the 90%CL region from $\mu \rightarrow e\gamma$ as obtained in simple models with approximate symmetry protecting neutrino masses $\varepsilon_{e\mu} = \sqrt{\varepsilon_{ee}\varepsilon_{\mu\mu}}$ [55, 95–102].

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